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Book Review: *Chaotic Dynamics of Nonlinear Systems*

Chaotic Dynamics of Nonlinear Systems. S. Neil Rasband, Wiley, New York, 1990.

Comparisons and classifications give a clearer insight into our understanding of our surroundings. The late L. D. Landau was one of the famous classifiers in science. "Only two physicists belong to the zeroth class—Newton and Einstein, who formed our world outlook," said Landau. "Those who created new sciences, such as Maxwell, Boltzmann, Dirac and Schrödinger, constitute the first class, while the creators of a new field in an existent science fall into the second class, etc." Landau was also a famous joker. Answering a provocative question concerning the famous scientist X whom Landau did not respect, Landau said, "To what class does X belong? I cannot see from here... He is too far away... Twenty six!"

This old anecdote comes back to my memory when I read the first sentence in the book under discussion: "Arguably the most broad based revolution in the world view of science in the twentieth century will be associated with chaotic dynamics." According to the author, chaos will have a broader influence on science than relativity and quantum mechanics.

Although this question is similar to asking who is stronger, the elephant or the whale, the very importance of chaos is beyond any doubt. Therefore there is room for many different textbooks on chaos, just as the shelves in our libraries are crammed with textbooks on analytical mechanics, electrodynamics, and quantum mechanics. The book under review is one in a series of such textbooks.

The introductory Chapter 1 contains a short description of the simple pendulum and kicked harmonic oscillator. It is seen from this analysis that one needs nonlinearity and at least three variables for the onset of chaos. The chapter concludes with a brief survey of applications of chaotic dynamics in physics, chemistry, biology, and engineering.

Chapter 2 includes the analysis of two one-dimensional maps, the tent and the logistic map. The former shows sensitive dependence on initial

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conditions as a result of stretching and folding operations, while the logistic map shows the set of bifurcations and Feigenbaum universal rules. A catalogue of possible bifurcations closes the chapter.

Chapter 3 continues the analysis of the logistic map, showing, for different values of the control parameter, the period-doubling and intermittency approach to chaos, as well as scaling, self-similarity, and subharmonic scaling of the logistic map.

Chapter 4 starts with a description of fractal geometry, descibing, in particular, four different dimensions of fractal systems. The fractal dimension of a strange attractor in dissipative systems is an example of the connection between chaotic dynamics and fractal geometry.

Chapter 5 includes an extension of the concept of chaos in discrete maps to differential equations. The linearization procedure is considered first, followed by invariant manifolds of linear flow, the center manifolds theorem and its applications, and the reduction to normal forms with different routes of bifurcations.

Chapter 6 expands the results of the previous chapter to the analysis of nonlinear differential equations. Three physical examples are considered in detail, namely, two coupled disk dynamos, the Lorenz system, and the driven pendulum with damping. The chapter concludes by outlining the third (in addition to period doubling and intermittency) possible route to chaos through quasiperiodicity and mode locking.

Chapter 7 brings us back to the maps—this time to two-dimensional maps. Some of their properties, such as bifurcations, normal forms, and asymptotic sets, are similar to those of the one-dimensional maps, whereas others, such as the Hopf bifurcations, and occurrence of resonances defined by the Arnold tongues, are inherent to two-dimensional maps. The Hénon map is considered in some detail.

Chapter 8 contains applications of area-preserved maps, considered at the end of the previous chapter, to conservative dynamical systems. Both completely integrable Hamiltonians (harmonic oscillator and pendulum) and nonintegrable Hamiltonians (the Toda lattice and nonlinear two-dimensional oscillator) are considered. Canonical perturbation theory is explained and applied to the perturbed twist map. Qualitative consideration of Arnold diffusion is carried out at the end of this chapter.

Chapter 9 raises the question of the measures of the chaotic nature of dynamical systems. The answer to the question, "How chaotic is chaos?," can be given by analyzing the Fourier spectrum, Lyapunov characteristic exponents, the Kolmogorov entropy as a measure of information, the generalized K-entropy, and the use of artificial phase-space dimensions.

Finally, Chapter 10 outlines the ways of making a distinction between deterministic chaos and randomness. Computational algorithms and complexity measure are described and illustrated by the example of the tent map.

This book can serve as the basis for a one-semester course for seniors or first-year graduate students of physics. It contains such typical features of a modern teaching course as execises, short summaries throughout the book, and references up to 1988. It was a very good idea to begin each new topic with references for more detail and deeper insight.

Of course, everyone has his or her own taste. I would prefer to see more detailed explanations of the renormalization procedure (Section 3.1) and bifurcations (Section 2.4) at the cost of abstract algebra and fractal dimensions. We know, however, from R. Kipling or, at least, from the dedication to one of the chapters of the Ziman book on solids, that "There are nine and sixty ways of constructing tribal lays, and-every-single-one-of-them-is-right!" In short, "let a hundred flowers bloom!"

In summary, I have no hesitation in recommending this book to the teacher to renew a traditional university program in science, or to the researcher interested in some special topics of the fascinating "world of chaos."

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